Clustering with Non-adaptive Subset Queries

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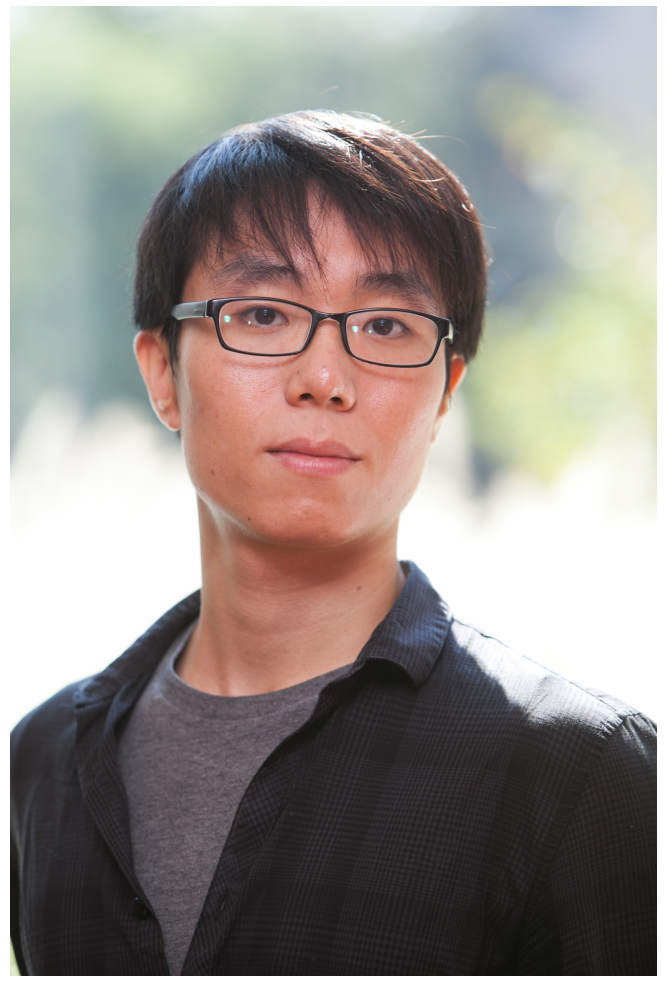
Hadley Black
UCSD

Euiwoong Lee
University of Michigan



Arya Mazumdar
UCSD

Barna Saha
UCSD

NSF UC San Diego Penn TEXAS UCLA ENCORE

Clustering via Crowdsourcing

Clustering: group data based on similarity

- Fundamental task in data science with many instantiations

Clustering via crowdsourcing:

- Can we offload the work of computing a clustering by asking simple questions to external individuals?
- **Same-cluster queries:** Are these two points of the same type?

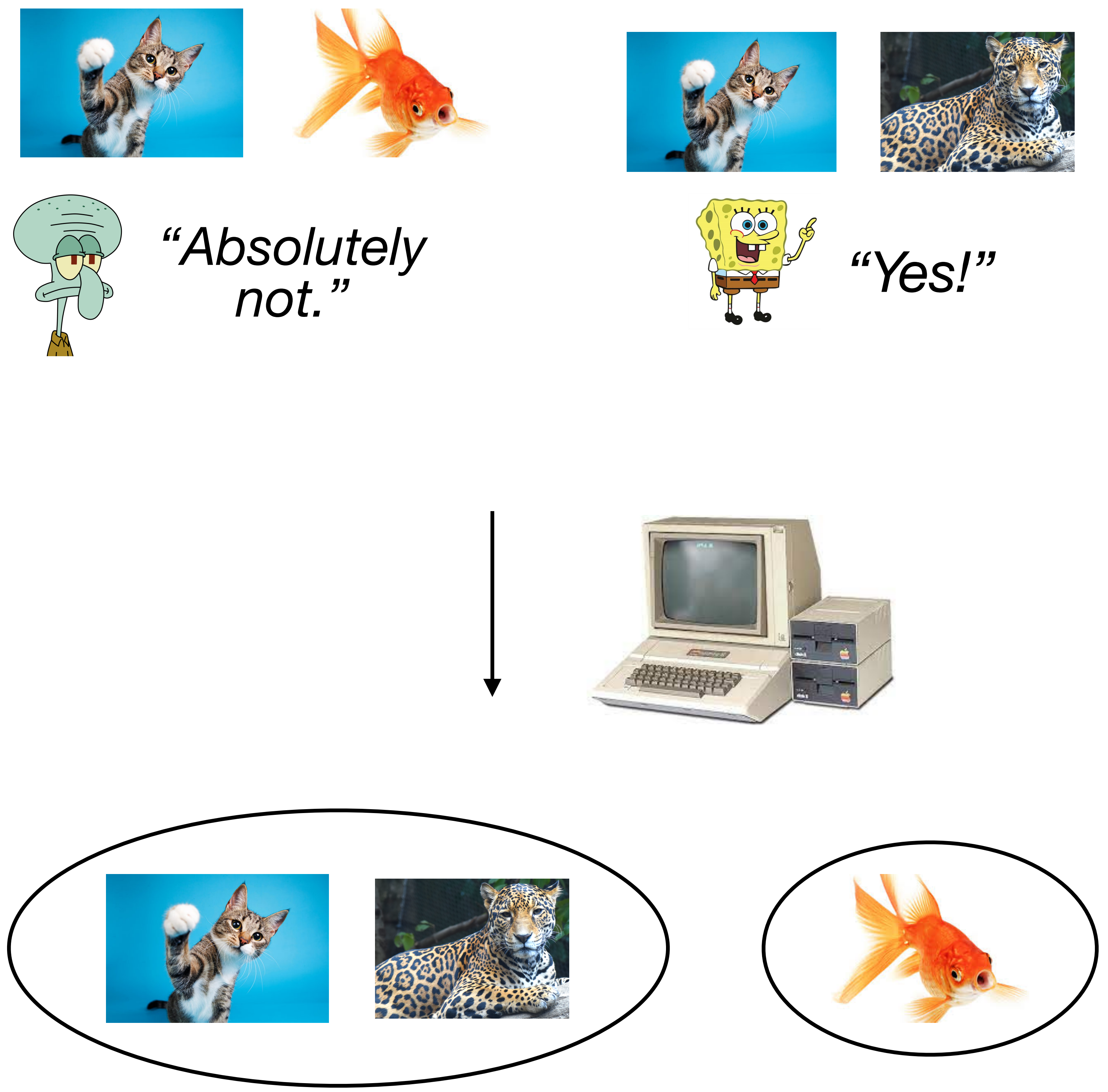
Wish list: (1) few queries, **(2)** queries specified in few rounds

- Spongebob & Squidward might be slow

  $\Longrightarrow$ Want to parallelize queries

- **Ideally**: non-adaptive (queries specified in one round)

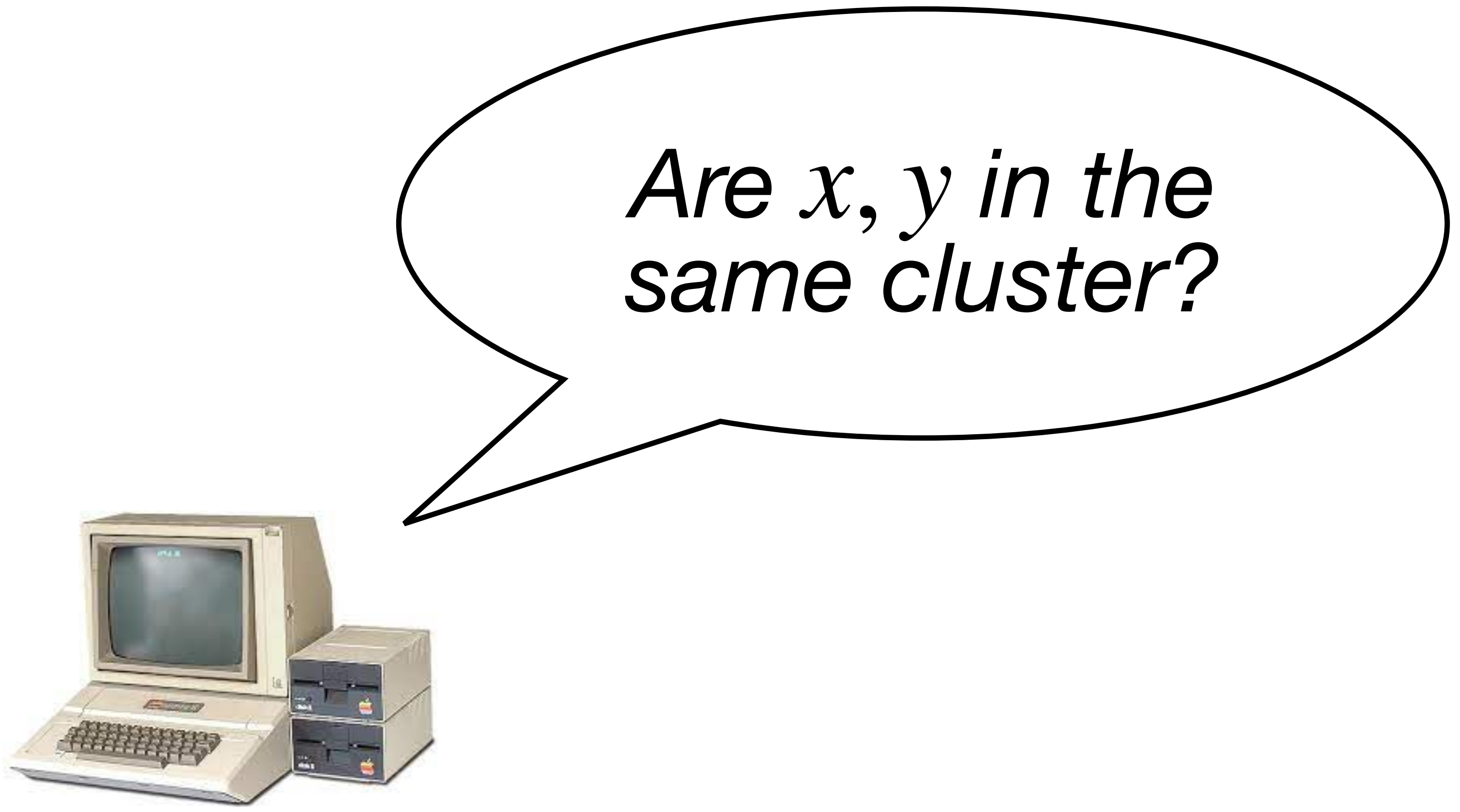
Query profile



Learned clustering

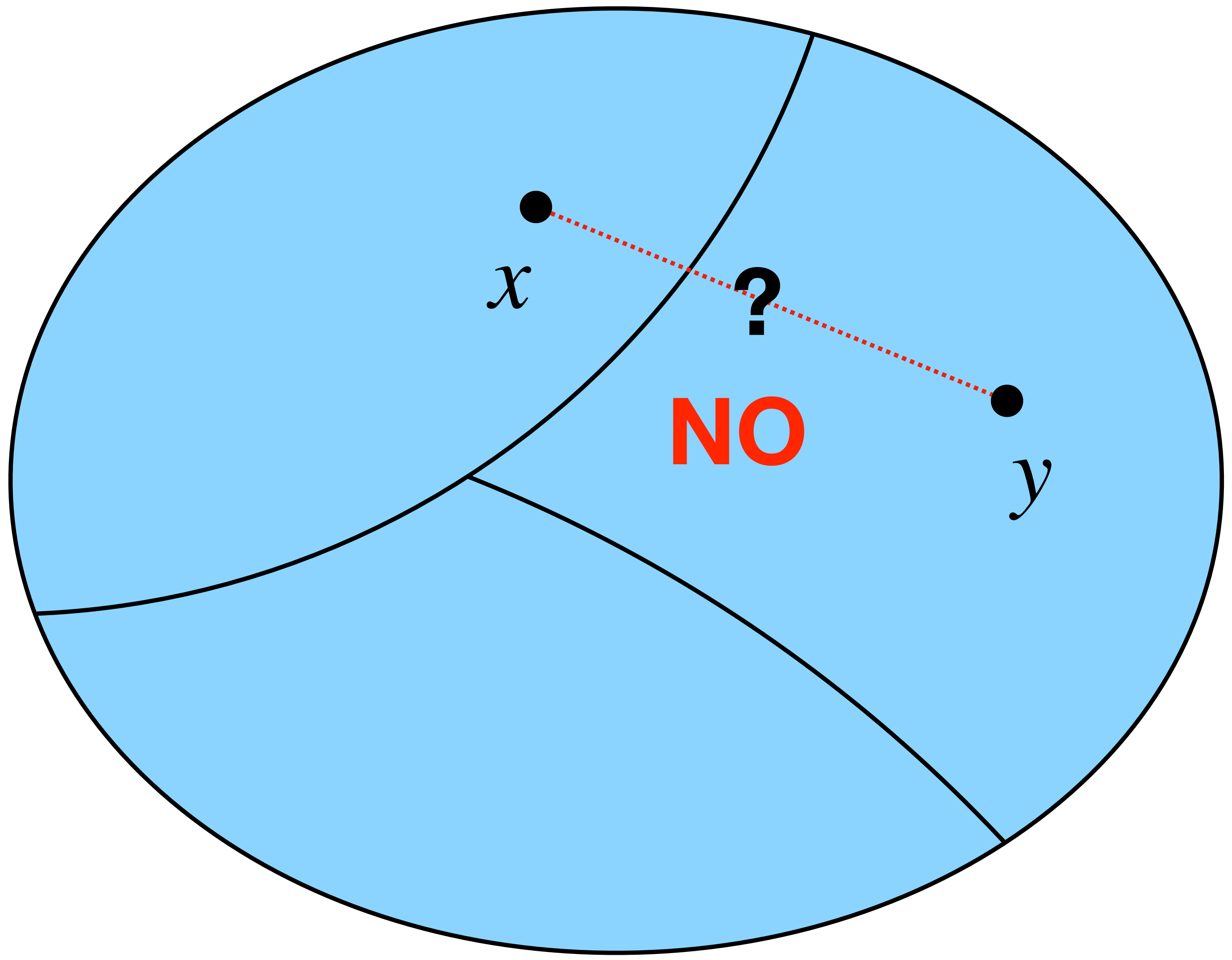
Clustering via Same-Cluster Queries

Mazumdar-Saha [Neurips 17], Mazumdar-Saha [AAAI 17], Mazumdar-Pal [Neurips 17], Mitzenmacher-Tsouraskis [16], Saha-Subramanian [ESA 19], Pia-Ma-Tzamos [COLT 22], Bressan-Cesa-Bianchi-Lattanzi-Paudice [Neurips 20], Huleihal-Mazumdar-Médard-Pal [Neurips 19]



- Set U of n points with hidden partition $C_1 \sqcup \cdots \sqcup C_k = U$
- Can **query** any $\{x, y\} \subset U$
    - Oracle says **YES** if x, y in same cluster and **NO** otherwise

Question: How many queries to learn $C_1, \ldots, C_k$ exactly?



Simple **adaptive** $O(nk)$ query algorithm ($k-1$ rounds) , **but…**

Theorem (MS 17, **BLMS 24)**

Non-adaptive algorithms require $\Omega(n^2)$ queries even for $k = 3$

$O(n^2)$ is trivial

Clustering via Subset Queries

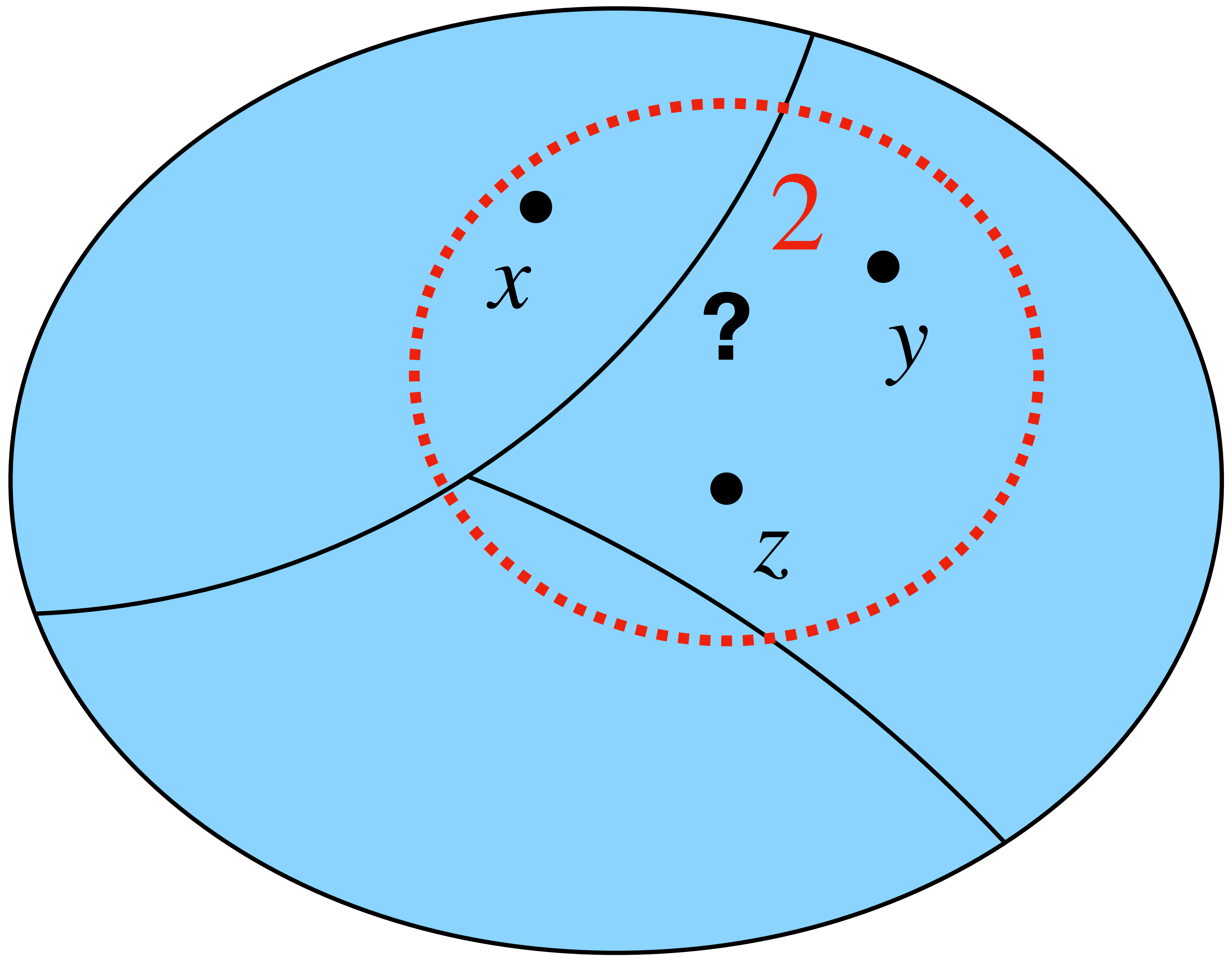
Chakrabarty-Liao [FSTTCS 24], Vinayak-Hassibi [NeurIPS 16] (considered triangle queries)

- Set U of n points with hidden partition $C_1 \sqcup \cdots \sqcup C_k = U$
- Can **query** any $S \subseteq U$ and oracle returns $\# j: S \cap C_j \neq \emptyset$

Question: How many queries to learn $C_1, \ldots, C_k$ exactly?

How many clusters does $\{x, y, z\}$ intersect?

Information-theoretic lower bound: $\Omega(n)$ $\Longleftarrow$ # bits per query: $O(\log k)$ # partitions: k^n



Theorem
(Chakrabarty-Liao 24)
$O(n)$ **adaptive algorithm**

Questions

How close to linear can we get non-adaptively?

How small of queries can we get away with?

Some of our Results (all algorithms and lower bounds are non-adaptive)

Unbounded subset queries

Theorem

$O(n \cdot (\log k + \log\log n)^3)$ for any k

$O(n \log\log n)$ for $k = O(1)$

Question

Is $O(n)$ for $k = 3$ possible using non-adaptive algorithms?

Subset queries of size $|S| \leq s$

Theorem

$O(n \log n \log\log n)$ for $s = O(\sqrt{n})$, $k = O(1)$

Getting near-linear requires $s = \Omega(\sqrt{n})$

Question

Can we get near-linear with $s = O(\sqrt{n})$ for all k?